the cases of $Z_1 > Z_2$, (experiments, part II), the loci of R are like open eccentric spirals about the center. The loci of sinusoidal tapers in Fig. 2 appear similar to a reduced concentric spiral about the center; these are typical for the cases of $Z_1 > Z_2$. The position of R also sweeps almost one cycle every half-wavelength. The locations in the K-plane and the forms of these loci are almost identical for several surge impedance ratios; hence these two figures depict typical characteristics for the general cases. Moreover, regarding the limiting cases of $l=0$ for linear tapers, I have obtained reasonable data for the behavior of R.

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The Permeability Matrix for a Ferrite Medium Magnetized at an Arbitrary Direction and Its Eigenvalues*

In analysis of propagation through magnetized ferrites it is usually assumed that the applied magnetostatic field is along one of the axes of the microwave carrier. It may be of interest to analyze the more general case; one in which the applied magnetostatic field is at an angle arbitrary to the axes of the microwave carrier.

If the geometry of Fig. 1, where $\boldsymbol{H}_0$ stands for the applied magnetostatic field and the carrier axes are x, y and z, is assumed, then, using the equation for the motion of the magnetization[1]

$$\frac{\partial \boldsymbol{M}}{\partial t} = \gamma \boldsymbol{M} \times \boldsymbol{H} \tag{1}$$

the following relation between the vector $\boldsymbol{B}$ and $\boldsymbol{H}$ results:

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \mu+(\mu_0-\mu)\sin^2\theta\cos^2\phi & \frac{\mu_0-\mu}{2}\sin^2\theta\sin 2\phi + j\kappa\cos\theta & \frac{\mu_0-\mu}{2}\sin 2\theta\cos\phi - j\kappa\sin\theta\sin\phi \\ \frac{\mu_0-\mu}{2}\sin^2\theta\sin 2\phi - j\kappa\cos\theta & \mu+(\mu_0-\mu)\sin^2\theta\sin^2\phi & \frac{\mu_0-\mu}{2}\sin 2\theta\sin\phi + j\kappa\sin\theta\cos\phi \\ \frac{\mu_0-\mu}{2}\sin 2\theta\cos\phi + j\kappa\sin\theta\sin\phi & \frac{\mu_0-\mu}{2}\sin 2\theta\sin\phi - j\kappa\sin\theta\cos\phi & \mu_0-(\mu_0-\mu)\sin^2\theta \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \tag{2}$$

or, in short notation,

$$\boldsymbol{B} = M\boldsymbol{H} \tag{3}$$

where the usual approximations of small signal theory[1] have been used. One may observe that this completely general permeability matrix still preserves Hermitian character as long as losses are neglected; and, of course, it reduces to simple well-known forms in cases when $\boldsymbol{H}_0$ is along any of the axes of the microwave carrier.

It may be sometimes desirable to express the relation between the vectors $\boldsymbol{B}$ and $\boldsymbol{H}$ in a canonical form. This can be accomplished by finding the principal axes of the medium —or, to put it differently—by finding a coordinate system in which there exists a relation of the form

$$(B_\alpha') = (\lambda_{\alpha\alpha})(H_\alpha') \tag{4}$$

where B_α' and H_α' are the components of the magnetic induction and magnetic intensity along the axes of the new coordinate system, and $(\lambda_{\alpha\alpha})$ is a diagonal matrix composed of the eigenvalues of the permeability matrix of (2). The procedure of finding the components of the matrix $(\lambda_{\alpha\alpha})$ is usually referred to as an eigenvalue problem.[2] In our case it amounts to finding an unitary matrix P such that

$$PMP^{-1} = (\lambda_{\alpha\alpha})$$
$$P^{-1} = \tilde{P}^*. \tag{5}$$

The transformation of the components of the magnetic induction and the magnetic intensity from the original to the new coordinate system are

$$B' = PB$$
$$H' = PH. \tag{6}$$

To find the matrix P we solve the eigenvalue equation

$$(M - I\lambda)U = 0 \tag{7}$$

where U is a matrix composed of three row vectors from which the matrix P can be constructed by means of an orthonormalization process.[3] Eq. (7) has a unique solution only if the determinant

$$|M - I\lambda| = 0, \tag{8}$$

which yields the results

$$\lambda_{1,2} = \mu \pm \kappa$$
$$\lambda_3 = \mu_0. \tag{9}$$

The eigenvalues of (9) are exactly the same as they would be if the applied magnetostatic field were along any one of the coordinate axes of Fig. 1. This fact may be somewhat surprising.

The amount of algebra involved in finding the matrix P corresponding to the permeability matrix of (2) is prohibitive. We shall try a simpler but still general enough case in which the applied magnetostatic field is in the $x-y$ plane, *i.e.*, $\theta=\pi/2$ in Fig. 1. In such a case the permeability matrix becomes

$$M = \begin{bmatrix} \mu+(\mu_0+\mu)\cos^2\phi & \frac{\mu_0-\mu}{2}\sin 2\phi & -j\kappa\sin\phi \\ \frac{\mu_0-\mu}{2}\sin 2\phi & (\mu+(\mu_0-\mu)\sin^2\phi & j\kappa\cos\phi \\ j\kappa\sin\phi & -j\kappa\cos\phi & \mu \end{bmatrix}. \tag{10}$$

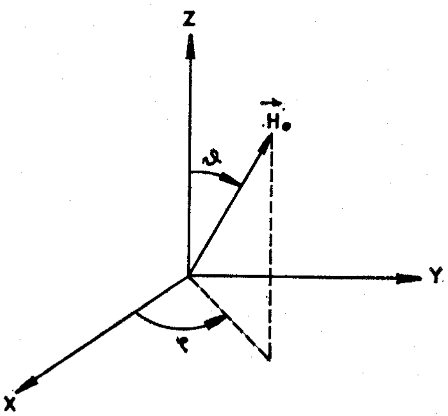

Fig. 1.

The corresponding unitary matrix P can be found to be

* Received by the PGMTT, September 12, 1958.

[1] D. Polder, "On the theory of electromagnetic resonance," *Phil. Mag.*, vol. 40, pp. 99–115; 1949.

[2] See, *e.g.*, H. Goldstein, "Classical Mechanics," Addison-Weseley Publishing Co., Inc., p. 119, 1956.

[3] H. Goldstein, *ibid.*, p. 328.

TABLE I

Original coordinate system	New coordinate system
$\nabla \times E = -\frac{\partial B}{\partial t}$	$\nabla' \times E' = -\frac{\partial B'}{\partial t}.$
$\nabla \times H = j + \frac{\partial D}{\partial t}$	$\nabla' \times H'^{*} = j' + \frac{\partial D'}{\partial t}$
$\nabla \cdot B = 0$	$\nabla' \cdot B' = 0$
$\nabla \cdot D = \rho$	$\nabla' \cdot D' = \rho'$
$B = MH$	$B' = (\lambda_{\alpha\alpha})H'$
$D = \epsilon E$	$D' = \epsilon E'^{*}$

Transformation equations

$$B' = PB$$
$$H' = PH$$
$$E' = -jP^{*}E$$
$$D' = jPD$$
$$j' = jPj$$
$$\nabla' = P^{*}\nabla$$
$$\rho' = -j\rho$$

$$P = \begin{bmatrix} j\frac{\sin\phi}{\sqrt{2}} & -j\frac{\cos\phi}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -j\frac{\sin\phi}{\sqrt{2}} & j\frac{\cos\phi}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\phi & \sin\phi & 0 \end{bmatrix}. \qquad (11)$$

We can now transform the entire set of Maxwell's equations to the new coordinate system which, we hope, will be simpler to work with. Table I shows the results.

It is indeed the case that the Maxwell equations—including the relations between the magnetic induction and magnetic intensity, and electric displacement and electric intensity—are much simpler in the primed than in the unprimed form. It may be an advantage in a particular problem involving arbitrary angle of magnetization of a ferrite to work in the primed system as far as possible before switching back to the original one.

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Resistive-Film Calorimeters for Microwave Power Measurement*

Two papers[1,2] published recently in these TRANSACTIONS have described calorimetric techniques for the measurement of microwave power at the milliwatt level which are free from the limitations inherent in existing methods using resistance-type milliwattmeters.

As the authors point out, the development of improved techniques is especially important at frequencies of the order of 10^4 mc and above. Somewhat similar work has recently been carried out in the United Kingdom at the Radio Research Station, Slough, and this note summarizes the essential features of the techniques used.

A 3-cm band calorimeter[3] in the form of a differential air thermometer has been developed for the power range 10–100 mw. This consists of two identical tapered resistive films located inside thin glass cells which are connected by a capillary tube containing a movable liquid index. One film absorbs the input microwave power and the other serves as a control against variations in ambient temperature. A measurement is made in terms of the equivalent dc power by a null technique. The input voltage standing-wave ratio (VSWR) is less than 1.15 over the band 8800–10,000 mc.

Comparison experiments have shown that the error limit is not more than ± 2 per cent. The instrument is extremely compact, and could be adapted for use at other frequencies.

Sucher and Carlin suggest that the substitution error in their calorimeters would be reduced to a minimum by using a transverse film as the absorbing load. This technique has in fact been used by the author for the measurement of power flow in rectangular waveguides, preliminary details being published in 1956.[4] A thin mica strip sputtered with platinum is located in the transverse plane, and with the optimum value of film resistivity an input VSWR of 1.1 or less can be obtained over the band 8500–10,000 mc, if the film is followed by a movable plunger. The input power can be determined in a dc calibration by utilizing the change of resistance produced in the platinum film, or, more conveniently, by observations of the temperature rise indicated by a thermocouple attached to the film. The wires of the thermocouple are parallel to the broad face of the waveguide. A detailed comparison[5] at 9200 mc, against reference standards operating at higher power levels has already confirmed that the technique affords a simple yet accurate method of power measurement in the range 1–100 mw.

Recent experiments (details of which are shortly to be published) have shown that this type of film bolometer can still be used at frequencies as low as 3000 mc. Furthermore, if a stable multirange dc amplifier is connected to the thermojunction, powers in range 100 μw–100 mw can be measured in a single instrument. Using this arrangement, the time constant of the 3-cm band model is not more than 5 seconds, compared with 2.6 minutes for the calorimeter described by James and Sweet.[2] The error limit is approximately ± 2 per cent at 100 mw and ± 5 per cent at 100 μw. Measurements at the latter level are at present limited in their accuracy as a result of random fluctuations in ambient temperature and amplifier gain. At a frequency of 10,000 mc, these fluctuations result in output variations equivalent to a power of about 3 μw. This "noise level" could probably be reduced by isolating the absorbing load in the manner described by James and Sweet.[2]

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[1] M. Sucher and H. J. Carlin, "Broad-band calorimeters for the measurement of low and medium level microwave power. I. Analysis and design," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 188–194; April, 1958.

[2] A. V. James and L. O. Sweet, "Broad-band calorimeters for the measurement of low and medium level microwave power. II. Construction and performance," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 195–202; April, 1958.

[3] A. C. Gordon-Smith, "A milliwattmeter for centimetre wavelengths," *Proc. IEE*, vol. 102, pp. 685–686; September, 1955.

[4] J. A. Lane, "A film radiometer for centimetre wavelengths," *Nature*, vol. 177, p. 392; February, 1956.

[5] J. A. Lane, "Transverse film bolometers for the measurement of power in rectangular waveguides," *Proc. IEE*, vol. 105, pp. 77–80; January, 1958.